

A.I.R.A.P. - Alternative RAPMs for Alternative Investments

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Abstract

This paper highlights the inadequacies of traditional RAPMs (Risk-Adjusted Performance Measures) and proposes AIRAP (Alternative Investments Risk Adjusted Performance), based on Expected Utility theory, as a RAPM better suited to Alternative Investments. AIRAP is the implied certain return that a risk-averse investor would trade off for holding risky assets. AIRAP captures the full distribution, penalizes for volatility and leverage, is customizable by risk aversion, works with negative mean returns, eschews moment estimation or convergence requirements and can dovetail with stressed scenarios or regime-switching models. A modified Sharpe Ratio is proposed. The results are contrasted with Sharpe, Treynor and Jensen rankings to show significant divergence. Evidence of non-normality and the tradeoff between mean-variance merits vis-à-vis higher moment risks is noted. The dependence of optimal leverage on risk aversion and track record is noted. The results have implications for manager selection and fund of hedge funds portfolio construction.

§1. Introduction

The heterogeneity of hedge fund strategies, their idiosyncratic bets, the complexity inherent in their dynamic trading and the extra degrees of freedom they possess (given the absence of leverage or shorting constraints), makes the task of judging managerial skill and performance particularly daunting. An increasingly popular alternative is to invest indirectly through fund of hedge funds (FoHFs hereafter). Liew (2002) suggests that, FoHFs with “good discernment,” can outperform their passively indexed counterparts. However, “good discernment” presupposes the existence of “good RAPMs¹.”

A flurry of recent papers such as Goetzmann et al. (2002), Spurgin (2001) and Bernardo and Ledoit (2000) have highlighted the inadequacies of traditional RAPMs such as the Sharpe ratio (hereafter SR). Alternatives and modifications to SR have been proposed, such as, Madan and McPhail (2000), Shadwick and Keating (2002a)² or Kazemi et al. (2003) while Leland (1999) proposes modifying the CAPM beta. In that vein, this paper introduces the proposed RAPM, AIRAP (Alternative Investments Risk Adjusted Performance), as the certainty equivalent. We follow the CRRA (Constant Relative Risk Aversion) framework of Osband (2002) but take the distribution-free route along the lines of CARA (Constant Absolute Risk Aversion) solutions by Davis (2001). This is the first paper to investigate the utility of certainty equivalence as a RAPM for hedge funds and to contrast the significantly different rankings obtained vis-à-vis SR and Jensen’s alpha (JA). Sharma (2003b) applies the AIRAP framework to re-visit empirical tests as well as to contrast hedge fund (HF) strategies at the index and fund levels.

This paper is organized as follows: §2 surveys traditional RAPMs. §3 reviews the expected utility framework and justifies our choice of the CRRA. A closed form, distribution-free solution for AIRAP is derived under CRRA. §4 analyzes the dataset used and presents rank correlations and reversals of competing RAPMs both at the strategy and hedge fund levels. Evidence of non-normality and the trade-off between mean-variance profile vis-à-vis higher moments is also examined. §5 investigates the impact of leverage on RAPMs while §6 proposes a composite ranking system for HFs based on AIRAP. §7 discusses caveats and concludes with thoughts on further research.

§2. Survey of RAPMs

While there exists no consensus on how to measure risk or risk-adjusted performance for HFs, the menagerie of RAPMs in circulation that could be applied includes SR, JA, Modigliani-Squared (M^2), M^2 -alpha, M^3 , SHARAD³, Treynor, Information Ratio (IR),

¹ Risk-Adjusted Performance Measures

² Shadwick, W. & Keating, C., 2002, “A Universal Performance Measure“, Journal of Performance Measurement, vol. 6, no. 3:59–84

³ The SHARAD (Skill, History and Risk-Adjusted) RAPM has been proposed by Muralidhar (2001/ 2002) as an extension of M^3 (see Muralidhar (2000)), since it explicitly adjusts for disparate performance history. Muralidhar, A., 2001, “Skill, History and Risk-Adjusted Performance“, Journal of Performance Measurement, Winter 2001/ 2002. Muralidhar, A., 2000, “Risk-Adjusted Performance – The Correlation Correction“, Financial Analysts Journal, 56(5):63-71.

Sortino, Calmar, Sterling, Gain / Loss, etc. Related performance statistics include Max Drawdown, # months to recovery, Peak-Trough, VAMI (value added monthly index), up / down market returns, upside / downside capture, etc. Finally, the associated risk metrics are beta, active risk, total risk, variance, semi-variance (upside/ downside & capture), MAD (Mean Absolute Deviation), VaR, VarDelta, Marginal VaR, CVaR (Conditional VaR or Expected Shortfall), CDaR (Conditional Drawdown at Risk), etc.

Absolute RAPMs consider portfolio returns in excess of the risk-free rate (viz., SR & Treynor) or zero (Calmar & Sterling). Relative RAPMs on the other hand, consider portfolio returns in excess of benchmark (Information ratio), Beta-adjusted benchmark (JA) or some threshold of minimum acceptable return⁴ (Sortino). SR & IR use the standard deviation of differential returns in the denominator to risk adjust, Treynor uses Beta, Sortino uses Downside Deviation (DD), Calmar uses Max Drawdown over 3 years and Sterling uses the average of Max Drawdowns over each of the past 3 years. Benchmark risk-equivalent RAPMs such as M^2 , M^2 -alpha, M^3 and SHARAD lever / de-lever portfolio performance in order to risk-equalize with the benchmark volatility, while borrowing at the risk-free rate. Since M^2 is an affine transform of SR it always produces the same rankings. Further, M^2 -alpha is a close cousin of Treynor and produces identical rankings. Hence, we do not dwell on either M-squared measures separately. M^3 was proposed by Muralidhar (2000) to augment M^2 by explicitly adjusting for benchmark correlation while SHARAD goes yet further by adjusting for disparate performance history (length of manager track records). Both M^3 and SHARAD differ from SR and are particularly germane to institutional benchmark relative performance measurement and risk-budgeting considerations. Despite the progressive institutionalization of hedge funds, correlation adjustment and tracking-error budgeting do not presently appear central to a class of investments still largely perceived as an absolute return class

The applicability and appropriateness of these RAPMs to HFs is a function of the efficacy of their associated risk measure in capturing HF risk. To the extent that *none* of standard deviation, Beta, downside deviation or Max Drawdown is a sufficient risk statistic *under non-normality*, *none* of the corresponding RAPMs should suffice for hedge funds. That said, each of these has its attractions worth highlighting. Calmar and Sterling are well suited for presenting the ‘worst case’ picture since they take into account Maximum Drawdown, i.e. the worst losing streak. Sortino, on the other hand, only adjusts by DD. The benefit is that DD does not penalize for upside variability but only for under-performance vis-à-vis some threshold of MAR. For predictably asymmetric returns, Sortino can be a better ex-post RAPM than SR since DD will in that case pick up on the realized skew and produce better portfolio rankings. Indeed, generalizing to the notion of lower partial moments (LPMs hereafter) can yield a host of new risk and corresponding risk-adjusted measures. The zeroeth LPM is just Shortfall risk or the frequency of under-performance vis-à-vis some MAR. The first LPM is just the mean under-performance, while the second LPM turns out to be Downside Variance.

⁴ MAR hereafter.

The Risk of RAPM shortfall:

Recent literature highlights the vulnerability of traditional RAPMs given the vagaries of leverage, non-normality and derivatives usage - issues which typify hedge fund returns. In the domain of risk measures, even the most popular candidates such as VaR fall short in that they cannot handle liquidity, credit or tail risk that are often characteristic of hedge funds. Further, VaR is not a “coherent risk measure”⁵ under non-normality, a deficiency that has led to the growing preference for Expected Shortfall (as a quantifier of tail risk), coupled with coherent scenario testing.

Amongst absolute RAPMs, we first consider Treynor. Treynor can be *magnified without bound*, via beta in the denominator. As a market neutral HF approaches beta-neutrality (as it should in order to uphold truth in advertising), Treynor approaches infinity. This is an issue for non-directional strategies in general, hence, Treynor is unsatisfactory for ranking / comparison of HFs.

Sortino performs a valuable function in adjusting by DD but can look deceptively high / favorable (upon trend reversal) if the ex-post estimation is based on a period of upwardly trending returns, since downside deviation underestimates the two-sided risk if the estimation period is not long enough to include loss periods. In this case, SR would perform better since standard deviation is not as vulnerable to a skewed sample when the underlying population is symmetrical.

Jensen’s alpha is *not leverage invariant*. Instead it scales in direct proportion with leverage thereby providing the perverse incentive to increase leverage without bound. In fact HF strategies, particularly relative-value strategies such as fixed-income or statistical arbitrage, are known to employ significant leverage in order to scale up their alpha. Assuming only IID returns for the market proxy, Leland (1999) shows that alpha can be systematically misguided because the CAPM beta ignores higher moments. Even if the single index CAPM world sufficed, Roll (1978) shows the arbitrariness of alpha rankings. If the benchmark used is mean-variance efficient, the securities market line is unable to discern out-performance. If not, then there exists some other non-efficient index which can reverse the ranking obtained.

Perhaps the most commonly used and widely respected RAPM in industry circulation is the SR. Sharpe has held the industry and academics in good stead since it was coined by its namesake and Nobel laureate in 1966. It has many desirable properties such as proportionality to the t-statistic (for returns in excess of zero) and the centrality of SR squared to optimal portfolio allocation.

However, SR is leverage invariant and it is not as intuitive as M^2 , M^3 and SHARAD which measure performance in basis point terms. It does not incorporate correlations nor can it handle iceberg risks lurking in the higher moments. Further, even the SR ratio can be ‘gamed’ by manipulating the returns profile. Spurgin (2001) has a recipe, which entails truncating the right tail. Similarly, Goetzmann et al. (2002) derive the optimal

⁵ Artzner et al., 1999, “Coherent Measures of Risk”, *Mathematical Finance*, 9 (3), 203-208.

static strategy via short OTM puts and calls which maximizes the SR ratio. This corresponds to a distribution with a truncated right tail (i.e. smooth monthly returns) but fat left tail (the periodic crashes). They remark, “the ‘peso problem’ may be ubiquitous in any investment management industry that rewards high Sharpe ratio managers.” Bernardo and Ledoit (2000) specifically demonstrate the limitations of the SR under non-normality. They show that outside the realm of normality, attractive investments (such as arbitrage opportunities) can have arbitrarily low SR ratios while poor investments may have high SR ratios. Although, given normality, SR suffices in completely characterizing investment desirability, “outside normality, it is impossible to make general statements that are preference-free other than no-arbitrage”⁶.

A number of cases manager hubris based on short but impressive track records (possibly attributable to short option profiles) have been documented. Jorion (2000)⁷ points this out for LTCM’s risk signature. Similarly, the well respected, Neiderhoffer, became victim to short OTM puts as a result of a sudden 7% market drop on October 27th, 1997. Lo (2001) observes, “Shorting deep out-of-the-money puts is a well known artifice employed by unscrupulous hedge-fund managers to build an impressive track record quickly”. More recently, Agarwal and Naik⁸ (2003) find the majority of HF strategies being characterized by short option profiles. They extend the previous findings of Mitchell and Pulvino⁹ (2001) for Merger Arbitrage. While there are isolated cases that put manager integrity into question, the broader issue remains one of investor suitability & whether that necessitates regulating access to hedge fund investments.

To assess suitability we must ask – Are hedge fund investors unwittingly underwriting disaster insurance unbeknownst to them? The operative principle in insurance is that risk transfer should result in the concentration of risk with the less risk-averse party. Arguably, the existing requirement of investor accreditation (which limits the audience to the less risk-averse) should assuage suitability concerns, although the emergence of vehicles that lower those requirements may not. The issue is germane to HFs not only because of their extensive use of derivatives and the option like characteristics of their incentive fees / survival likelihoods, but also because they are often marketed on the basis of RAPMs such as the SR. To the extent that financial intermediaries (such as registered brokers) are sufficiently trained to assess suitability and investors sufficiently aware of risks (when viewed as ‘stand-alone’ investments) then the focus can shift to the potential portfolio level benefits of adding HFs based on marginal risk-return characteristics.

Bookstaber and Clarke (1981) show that options can skew portfolio returns distribution, rendering the mean-variance framework inadequate. Fama (1976) provides evidence of leptokurtosis for individual stocks¹⁰. While market participants may be anecdotally or

⁶ Bernardo & Ledoit (1996), p. 7

⁷ Jorion, Philippe, 2000, “Risk Management Lessons from LTCM”, European Financial Management.

⁸ Naik, N.Y. & Agarwal, V., 2003, “Risk and portfolio decisions involving hedge funds”, Review of Financial Studies, forthcoming.

⁹ Mitchell, M. & Pulvino, T., 2001, “Characteristics of Risk and Return in Arbitrage”, The Journal of Finance, Vol 56, Issue 6: 2135-75.

¹⁰ On the other hand, Koski & Pontiff (1996) find that the use of derivatives in mutual funds does not significantly alter either risk or return profile. However, derivatives do significantly alter kurtosis for

peripherally aware that the crash of 1987 was an ab-'normal' 27 sigma event, they may perceive the relevance of higher moments as merely an esoteric concern. Table 1 shows negative skew and excess positive kurtosis for the S&P 500 over each of the trailing 10, 20, 30 and 40 year periods¹¹ (02/1962-01/2002). Goodness of fit tests for the past 40 years confirms our suspicion. Table 3 shows rejections of normality by Bera-Jarque at the 99% level and by Lilliefors at the 95% level. Figure 1 and Table 6 show for the HF case that the mean, median and mode of skewness are all negative as is the skew of skewness. Worse yet, Figure 2 and Table 6 show that the mean, median and mode of kurtosis (in excess of normality) are significantly positive with positive skew and kurtosis of kurtosis.

Sharpe (1994) himself remarks, "comparisons based on the *first two moments* of a distribution do *not take into account possible differences* among portfolios in other moments or in distributions of outcomes across states of nature that may be associated with *different levels of investor utility*."¹² Clearly then, what is needed is a measure that incorporates investor preferences via risk aversion and that which adjusts for iceberg risks lurking in the higher moments.

Finally, SR is plagued by another deficiency, which limits its utility during bear markets. Table 5 uses the EACM Bond Hedge index (1997-01) to show that a fund with the same excess return (-4.5%) but twice the risk (12%) has an SR twice as good instead of its being twice as worse. This happens because -0.37 is larger than -0.74 even though smaller in absolute magnitude.

§3. Expected Utility theory and AIRAP:

Expected Utility theory is central to the foundations of modern economics and dates back to the axiomatization of Von Neumann–Morgenstern (1944). Under Transitivity, Completeness, Independence and the Archimedean axioms, investor preferences have an Expected Utility representation (which is unique up to affine transforms). The Expected Utility property allows for the expression of the Von Neumann–Morgenstern utility U of a lottery (with payoffs z_1 and z_2 and probabilities p and $(1-p)$) as $[p.u(z_1) + (1-p).u(z_2)]$. Here $u: x \rightarrow \mathbb{R}$ is the real valued Bernoulli utility, which is a function of payoffs (as opposed to lotteries for U). Allowing a lottery to be represented by the real-valued random variable w enables us to use the fact of U representing preferences over lotteries to be equivalently stated in terms of preferences over cumulative distributions F :

mutual funds, but the bias is not systematic (presumably because they are also being used for hedging and to reduce returns variability which would decrease kurtosis). The situation with hedge funds appears to be quite different. Koski, J. L. & Pontiff, J., 1996, "How are Derivatives Used? Evidence from the Mutual Fund Industry", working paper series, Wharton Financial Institutions Center.

¹¹ The effect would be more pronounced with daily instead of monthly data.

¹² The italicization is mine.

For lotteries w & w' : $F_w \geq F_{w'} \equiv U(F_w) \geq U(F_{w'})$, where $U(F_w) = \int_R u(x) dF_w(x)$

For RAPMs to be useful they should incorporate risk aversion since the investor expects to be paid a risk-premium for owning risky assets. In this case risk aversion is embodied by the concavity of u (risk-proclivity by convexity and risk-neutrality by linearity).

Given the concavity of u and Jensen's inequality we obtain¹³:

$$u(E(z)) = u(pz_1 + (1-p)z_2) > pu(z_1) + (1-p)u(z_2) = E(u)$$

Risk aversion is captured by $u(E(z)) > E(u)$, since the utility of the mean but certain payoff $E(z)$ exceeds the utility of the uncertain lottery (with payoffs z_1 and z_2). The *certainty equivalent* lottery can thus be defined as that lottery which pays $CE(z)$ and has the same utility as the uncertain lottery (Figure 6), i.e. $u(CE(z)) = E(u)$. Risk-aversion qua concavity entails that the payoff $CE(z) < E(z)$. Hence, the Risk-Premium is defined as,

$$RP(z) = [E(z) - CE(z)] \text{ where } RP(z) \geq 0$$

In the RAPM world, $CE(z)$ is the *implied equivalent return* that the risk-averse investor desires with *certainty* in exchange for the *uncertain return* from holding *risky assets*. Consequently, $RP(z) \geq 0$ (under risk-aversion) is the price paid for trading off the risky asset with $CE(z)$. Hence, certainty equivalence provides an intuitive risk adjustment for our definition of AIRAP. By stripping out varying risk premia earned, it facilitates a fair comparison of HF performance. Further, strict monotonicity and continuity¹⁴ of the chosen utility function ensure invertibility, resulting in $CE(z)$ rankings that are identical to maximizing $E(u)$, since by definition, $CE(z) = u^{-1}(E(u))$.

We now proceed with choosing the appropriate form of utility. The standard mean-variance framework is justified either on the basis of tractability of quadratic utility (for arbitrary distributions) or multivariate normality (for arbitrary preferences). However, neither assumption is satisfactory. Quadratic utility displays satiation¹⁵ and IARA (Increasing Absolute Risk Aversion), since it views risky assets as inferior goods. Nor does it not show skewness preference while normality is a poor assumption for HFs (table 7). In fact, HF data does not even usually satisfy the premises of the central limit theorem (see Getmansky et al. (2003)).

A CRRA formulation has the benefit of being impartial to wealth level (which is to be expected of asset managers from a fiduciary perspective), excluding negative wealth (consistent with hedge fund losses under limited liability being bounded below by the principal invested) & *scale invariance*. For instance, a high net worth family in the top

¹³ EU measures are complete, transitive & convex like the underlying preferences.

¹⁴ This follows from the inverse function theorem. Continuity & existence of the inverse makes the original function a homeomorphism.

¹⁵ The assumption is that rational investors prefer more to less and view risky assets as normal goods. See Huang, C. & Litzenberger, R., 1988, *Foundations for Financial Economics*, Prentice-Hall, NJ.

39.1% tax bracket¹⁶ will be relatively indifferent to a \$10,000 loss (< 3.4% of income) vis-à-vis a family at the poverty line¹⁷ for whom the same loss amounts to 55.3% of income. See Osband (2002) for an exposition of utility functions and the relative merits of CARA vs. CRRA.

The best known measure of risk aversion is Arrow-Pratt, which quantifies the normalized magnitude of concavity of u times wealth via its 2nd derivative. Assuming concavity, monotonically increasing and twice differentiable utility, CRRA is tantamount to requiring that the Arrow-Pratt RRA coefficient is a constant c :

$$-\frac{u''(w)}{u'(w)}w = c$$

We assume the NAV (net asset value) process represents HF total returns (since dividends are rare),

$$\%TR_t = (NAV_t - NAV_{t-1}) / NAV_{t-1}$$

CRRA utility corresponds to the family of power utility functions defined for terminal wealth, W_T as,

$$U(W_T) \equiv \frac{W_T^{(1-c)} - 1}{(1-c)}, c \neq 1, c \geq 0 \text{ and } U(W_T) \equiv \ln(W_T) \text{ when } c = 1$$

Given that terminal wealth is just the initial wealth compounded at %TR,

$$W_T = W_0 * (1+TR)$$

and CRRA rankings are scale invariant, then $U(W_T)$ rankings are the same as $U(1+TR)$.

$$U(1+TR) \equiv \frac{(1+TR)^{(1-c)} - 1}{(1-c)}, c \neq 1, c \geq 0 \text{ and } U(1+TR) \equiv \ln(1+TR) \text{ when } c = 1$$

For a finite discrete distribution¹⁸ (true for any histogram of empirical returns), we can now solve directly for CRRA CE from $EU=U$, albeit not explicitly parsed in terms of higher moments. Let p_i represent the probability of the i^{th} return of N observed $TR_i, i=1, \dots, N$, such that:

¹⁶ Top marginal tax rate of 39.1% for Head of Household kicks in at \$297,350 threshold (2001 IRS tax schedule).

¹⁷ For a 4-person family the poverty line is defined as \$18,100 in 2002 (cf. *Federal Register*, Vol. 67, No. 31, February 14, 2002, pp. 6931-6933).

¹⁸ This derivation ignores subtleties such as rebalancing, transaction costs, etc.

$$\begin{aligned}
EU &\equiv \sum_i \left[\frac{(1+TR_i)^{(1-c)} - 1}{(1-c)} \right] \cdot p_i = U \Leftrightarrow \left[\sum_i p_i \cdot (1+TR_i)^{(1-c)} - \sum_i p_i \right] = (1+CE)^{(1-c)} - 1 \\
&\Leftrightarrow \left[\sum_i p_i \cdot (1+TR_i)^{(1-c)} \right] = (1+CE)^{(1-c)} \Leftrightarrow \\
AIRAP = CE &= \left[\sum_i p_i \cdot (1+TR_i)^{(1-c)} \right]^{\frac{1}{1-c}} - 1, \quad \text{when } c \neq 1 \& c \geq 0
\end{aligned}$$

To avoid restrictive or questionable distributional assumptions, one can now proceed with any one of many available non-parametric estimation techniques. We emphasize the generality of this result, since the choice of non-parametric method is a matter of taste, and the resulting AIRAP estimate need not be tied to it. Still it is worth highlighting a particularly simple solution that results from fitting a histogram where $p_k = [\text{frequency of \%Returns in the } k^{\text{th}} \text{ bin} / N]$, $k=1, \dots, M$. Since arbitrariness in the choice of bin size results in arbitrariness of M and precision of the AIRAP estimate, we set the bin width¹⁹ as:

$$\varepsilon := \frac{1}{2} * \text{Min}\{|TR_i - TR_j|\}, \quad \forall i \neq j$$

Starting with the leftmost observation, the ε -bins are centered on each TR_i such that all distinct TR_i fall in exactly one bin. Thus, for all non-empty bins, $p_i = 1/N$. Substituting with $1/N$ ²⁰ for p_i in AIRAP yields a convenient closed form simplification.

When $c=1$, one proceeds in a similar manner to solve for AIRAP under log utility:

$$\begin{aligned}
EU &\equiv \sum_i \ln(1+TR_i) \cdot p_i = U \equiv \ln(1+CE) \\
&\Leftrightarrow \ln \left\{ \prod_i (1+TR_i)^{p_i} \right\} = \ln(1+CE) \\
&\Leftrightarrow AIRAP = CE = \left[\prod_i (1+TR_i)^{p_i} \right] - 1, \quad c = 1
\end{aligned}$$

Again setting $p_i = 1/N$, provides a closed form solution that has a straightforward spreadsheet implementation. In general, any non-parametric estimate as outlined above has the dual benefit of being distribution free and of capturing all observed moments. Note that an analogous derivation is obtainable under exponential utility (CARA), which

¹⁹ This could be called the degenerate histogram method. The end result resembles the L_p norm, for $p=(1-c)$, except that $p \neq 0 \& p \leq 1$.

²⁰ This works even if one occasionally encounters k returns that are identical since the frequency in the overlapping bins simply add up to k/N , the probability assigned to that TR_i at the mid-point.

would be a special case of the closed form solution in Davis (2001) for histograms²¹. Hence, AIRAP could be formulated for CARA with ease. For comparison, we note that Madan and McPhail (2000) as well as Davis (2001) use exponential utility while Osband (2002) and Leland (1999)²² use power utility.

Recommended Arrow-Pratt coefficient:

For power utility, $c > 0$ represents risk-aversion. When $c = 0$, $U(TR)$ is linear in %TR and AIRAP is simply the arithmetic mean or in the annualized case it is the geometrically compounded monthly arithmetic mean excess return. For $c = 1$, logarithmic utility results in AIRAP as the geometric mean of monthly excess returns²³. Since $0 < c < 1$ implausibly allows rational investors to entertain bets potentially resulting in insolvency, we restrict our attention to $c \geq 1$. In the latter case, the pain of insolvency is unbounded, precluding bets that could risk total ruin.

The plausible range for c is from 1 to 10. Osband (2002) suggests using c from 2 to 4. Ait-Sahalia et al. (2001) propose a resolution to the equity premium puzzle by examining data on the consumption of luxury goods²⁴ by the very rich who also constitute the majority of equity ownership. Their point estimate of $c = 3.2$ (s.e. 2.2) for ultra high net-worth individuals²⁵ seems most pertinent to HF investors. To be quite conservative we assume $c = 4$, in which case the CRRA agent is willing risk no more than a fifth of her wealth for even odds of doubling.

The dependence of this approach on parameter c may be viewed as undesirable from a practical standpoint, given the ongoing academic debate over the true value of c and its implications for the equity risk premium puzzle. However, for RAPM purposes this is not an impediment. As long as we can target a plausible but fixed c , the ranking of all funds under AIRAP will be comparable and consistent. There is a *possible significant benefit* to the *flexibility* of being able to tweak risk-aversion. Technology can enable financial advisors/ investment managers to query data on investor risk preferences and map them to an individualized c , thereby generating *customized AIRAP rankings*.

²¹ Davis (2001) also has CE solutions for exponential utility (but none for power utility) under a host of other distributions.

²² Leland (1999) only assumes IID returns for the market proxy but given 'perfect' markets in his framework, it turns out that the representative investor must have power utility.

²³ Given our use of the geometric mean (annualized) in measuring average performance, $RP = 0$ corresponds to $c = 1$. If instead, we were to use the geometrically compounded (annualized) monthly arithmetic mean, then $c = 0$ would correspond to $RP = 0$. However, for long horizons the latter diverges significantly from the average annualized performance a fund investor would obtain.

²⁴ Their contention is that NIPA and household survey data used in prior literature (on basic goods consumption) overstates risk aversion by an order of magnitude.

²⁵ This estimate of RRA is implied from 'Luxury Retail Sales (US Retailers).' For perspective, they also find that the c implied by 'Charitable Contributions of Rich' is 4.7 (s.e. 3.3).

§4. Data & Analysis

We use 5 years of monthly data (01/97 to 12/01)²⁶, for the EACM (Evaluation Associates Capital Markets) indices for our index level analysis, since these indices are recognized for their style pure categorization. The EACM100 is an equally weighted, annually re-balanced composite of 100 funds rigorously screened to represent 5 strategies (13 style sub-indices). It has adequate data history (extending to 1996) and does not allow closed funds. At the individual fund level (where EACM does not disclose constituents), we resort to the HFR²⁷ universe given its wide usage and recognition for lower survivorship bias. Of the 2445 entries in HFR as of June 2003, only 887 HFs existed for the entire 5-year period. 100 time series corresponding to HFR indices were excluded. The final 787 HFs include on and offshore funds, FoHFs, managed futures, as well as sector HFs.

Our data set is long enough to be meaningfully subject to analysis, without being too long to be afflicted by more survivorship bias. Further, the choice of this period was motivated by the desire to include the Asian crisis (1997), the Russian crisis and LTCM debacle (1998), the bubble era (through 1999) and the subsequent Nasdaq collapse. We do not explicitly adjust for survivorship, instant history, selection and other well-known biases, since the objective is to study *relative* rankings. Table 4 shows the aggregate statistics for the 1st four moments and various RAPMs with regard to the HFR universe. This in conjunction with Figure 2 shows the distribution of ExKurt to be right skewed (+4.54) with a long right tail given a max of 51.4. Average ExKurt of 3 is significantly non-Gaussian with over 87% of funds showing positive ExKurt. The mean skewness is -0.14, while the skewness of skew is also negative (-1.24). This could have been worse at the composite level if not for the counterbalancing effect (Table 5) of Managed Futures and Macro funds in the sample.

We display rank correlations and reversals between SR, JA and power utility (AIRAP) for the full HFR universe of 787 funds, as a function of c (between 0.1 and 30), in Table 10. RAPM ranks and correlations for the 13 EACM style sub-indices appear in Table 5. The SR rank correlations (Table 10) are similar to Fung and Hsieh (1997) except that their study used 233 funds, defined SR in terms of total not excess returns and did not look across style categories and databases. More importantly, their objective was to check for the near sufficiency of mean-variance in portfolio construction as opposed to the suitability of RAPMs for HFs.

The performance of SR in ranking hedge funds is significantly misleading with respect to the investor's true utility rankings as per both ranks reversals and correlations (Table 10). Pearson correlations (Table 6) are even weaker at .46, .37 & -0.01 for SR, JA & Treynor. Our correlations (Table 10) are similar to Fung and Hsieh (1997) for CRRA in the range [3, 5] but theirs drop off much faster for lower values of CRRA while ours decline faster for higher levels of risk aversion. Further correlations of AIRAP with Alpha decrease dramatically with increasing risk aversion. This may be explicable since AIRAP imposes

²⁶ It is best to use data post 1994 since most databases exhibit severe survivorship biases prior to 1994.

²⁷ Hedge Fund Research, Inc., © HFR, Inc., www.hedgefundresearch.com

a steeper risk penalty, as an increasing (but non-linear) function of risk-aversion while Alpha is invariant with regard to risk-aversion.

Scatter plots of RAPM rankings (Figure 3) with the abundance of off-diagonal funds visually confirm the noted lack of correlation and that the picture is essentially the same for CRRA in the 2 to 4 range. Treynor with the lowest AIRAP correlation of 0.49, erroneously penalizes funds with slight negative beta exposures or negative means, resulting in the cluster to the south-east corner. Alpha on the other hand (+.66 correlation), creates a cluster to the north-west comprised of funds that are in most cases either negative beta or where the CAPM beta fails to capture risk. Short sellers are grossly misrepresented due to their negative betas, resulting in JA being artificially boosted and Treynor being inappropriately depressed. Table 9 of representative funds shows that #512 has the worst AIRAP rank (1) even though SR ranks it as 133 (of 787), because not only are returns low (10) and 56% Vol extreme (786 rank) but iceberg risks are high (ExKurt is 682 while Skew rank is 58). On the other hand, #235 has the worst SR (1) due to negative mean and low Vol (8). AIRAP correctly handles the negative mean and boosts the rank by 201 notches since the higher moments are tame & Vol exceptionally low. Fund #229 has the highest Alpha (787 rank due to negative beta), middle of the pack SR (482) but AIRAP is 775 notches lower because of the penalty for extreme 83.2% Vol. For EACM sub-indices, AIRAP penalizes on average 2% more than JA does. It is systematically lower than Alpha for all but Event Driven sub-indices. In the case of Multi-Strategy Relative Value, the penalty is 1.2% more largely due to the -4.6 skew and ExKurt of nearly 25.

To show that AIRAP conveys new information not already captured by traditional RAPMs, we show Spearman rank and Pearson correlations²⁸ in Table 6. For the HFR universe, AIRAP is positively correlated with ExTR, Skew, Treynor, Jensen & SR but negatively correlated with Volatility & Beta as one would expect. To the extent that a large dispersion in mean-variance profiles has been documented across strategies and these effects often dominate higher order effects, one should expect drastic rank reversals for the full HFR universe that aggregates across strategies. We find that *% rank reversals are in excess of 99% across the board*, i.e., at the broad universe level, there is almost no agreement between AIRAP and Sharpe or Alpha rankings. This observation needs to be tempered by the realization that for a given rank order $\langle 1, 2, \dots, 787 \rangle$ the trivial permutation $\langle 2, 3, \dots, 787, 1 \rangle$ results in 100% reversals despite near-perfect correlation. The key is that the magnitude of some of these reversals (in addition to their prevalence) can be substantial as per Figure 3, anecdotal evidence above and the rank correlations previously noted.

Rank discrepancies at the intra-strategy level are likely to be fewer if HFR strategies are sufficiently style-pure to reduce heterogeneity. While the 90% average reversal rate from intra-strategy rankings in table 7 is somewhat lower, it is well known that the self-proclaimed style of managers in databases such as HFR need not be a reliable indicator of

²⁸ Rank correlations are to be preferred in assessing the co-dependence since the data suggests non-linear dependence rendering Pearson correlation less appropriate.

the factors they load on. The magnitude of intra-strategy rank discrepancies and how that relates to the aggregate level across databases is further documented in Sharma (2003b).

Strategies with higher iceberg risks like HFR Merger Arbitrage and Event Driven seem to have higher reversal rates (Table 7) than more liquid strategies like Shorts (82%). Short Sellers do display a much lower incidence of non-Gaussian profile (36%) as compared to Event Driven (80%). Indeed, our results for EACM sub-indices (table 5) show Event Driven as well as Relative Value & Event Driven multi-strategy being demoted a notch under AIRAP. Liquid equity strategies with controlled volatility such as Domestic Opportunistic, Global and Long/ Short do move up 2-4 notches. It would therefore appear that style categories exhibiting greater departures from normality (i.e. higher moment risks) also exhibit greater rank discrepancies between SR & AIRAP. However, the picture is muddled by the complex interaction of volatility with higher moments (since manifestation of higher kurtosis can percolate into volatility / skewness and vice-versa) and the fact that the higher magnitude volatility penalty often dominates. E.g., high volatility (despite innocuous higher moments) results in AIRAP severely penalizing Long Biased (RP=7.37%) and Short EACM sub-indices (RP=6.9%). This interaction is often easier to disentangle at the individual fund level than at the aggregate category level.

The related claim - *high Sharpe ratios in hedge funds may represent a trade off for higher moment risk* – is investigated in Sharma (2003b). Here we simply note the positive (and statistically significant) rank correlation of SR with excess kurtosis for both EACM and HFR data (tables 5 and 6). To the extent that some HF strategies pay for a better mean-variance profile by assuming iceberg risks, it seems less plausible that they are better exploiting inefficiencies or expanding the investment opportunity set. At least part of their *mean-variance attraction may stem from the pre-meditated but potentially suicidal (short volatility) act of scooping up pennies before the onslaught of the steamroller.*

Scott and Horvath (1980) show that *risk-averse investors prefer positive odd central moments (such as skewness) and dislike even central moments (such as kurtosis)*. Unlike traditional RAPMs (which are largely oblivious to the impact of higher moments), AIRAP critically *penalizes for negative skew and positive kurtosis.*

§5. Impact of Leverage

Traditionally, the leverage invariance of SR has been considered desirable. This makes sense for traditional investments since leverage is neither central to the investment strategy nor usually permissible under existing regulation (e.g., with mutual funds). If used at all, leverage is usually employed by means external to the core investment vehicle, perhaps at the allocation level or through structured products.

Leverage to the hedge fund manager is a critical extra degree of freedom, especially for relative value / arbitrage strategies. The decision, whether to use leverage and to what extent is integral to the hedge fund investment process. The impact of leverage on the

realized distribution should not be ignored²⁹. For ranking and comparison purposes, either we must use un-levered returns or account for leverage directly. Given the lack of transparency with HFs, computing un-levered returns may be impractical. Besides, investor utility is a function of the realized total return achieved not some hypothetical un-levered return which may have been achieved had the manager not made the wise or unwise decision to use a given degree of leverage. Hence, appropriately accounting for leverage requires accommodating preferences, i.e., a good hedge fund RAPM *should encapsulate aversion to excessive leverage under risk-aversion*.

To understand how AIRAP incorporates leverage, we consider only financing leverage, i.e. the impact of levered exposure to the same risky fund enabled through borrowing. This suffices since AIRAP already adjusts for the market risk of the underlying fund based on returns data. Table 8 shows the impact of leverage on EACM 100. We assume that n-times leverage corresponds to the excess return scaled up by n, since the differential return is a self-financed portfolio. Hence, the mean monthly excess return of 0.40% doubles to 0.80% for 2x and rises to 6% for 15x leverage. Volatility, Beta and Alpha rise also linearly by exactly the leverage factor n. Since Alpha rises in proportion to leverage, it is inappropriate for HFs as it indiscriminately rewards higher leverage without bound. The proportional rise in Beta does not sufficiently penalize for the rise in volatility under risk aversion, even though skew and excess kurtosis are unchanged. Sharpe and Treynor on the other hand are leverage invariant³⁰. They are oblivious to the impact of leverage since the 1st & 2nd moments³¹ rise in tandem and cancel out.

$$Sharpe_{P, Levered} = \mathbf{m}_{P, Levered} / \mathbf{s}_{P, Levered} = n * \mathbf{m}_p / n * \mathbf{s}_p = Sharpe_p$$

$$Treynor_{P, Levered} = \mathbf{m}_{P, Levered} / \mathbf{b}_{P, Levered} = n * \mathbf{m}_p / n * \mathbf{b}_p = Treynor_p$$

$$\mathbf{a}_{P, Levered} = R_{P, Levered} - \mathbf{b} * R_B = n * (R_p - \mathbf{b} * R_B) = n * \mathbf{a}_p$$

$$\mathbf{b}_{P, Levered} = \mathbf{r} * \frac{\mathbf{s}_{P, Levered}}{\mathbf{s}_B} = \mathbf{r} * \left(\frac{n * \mathbf{s}_p}{\mathbf{s}_B} \right) = n * \mathbf{b}_p, \because \mathbf{s}_{P, Levered} = n * \mathbf{s}_p$$

AIRAP penalizes for increased leverage as a function of risk-aversion. The impact of leverage on the returns distribution is captured via credit for the higher mean and penalty for the higher volatility as a function of the CRR parameter. E.g., In going from 5x to 10x, RP jumps by 46.4%, from 12.3 to 58.7% (CRR=4), turning AIRAP negative (-18.9% despite +39.8% Excess TR) in Table 8. The alpha of 37.3% and static 0.90 SR would have misled us in this instance. Assuming lower risk aversion e.g., CRR=2,

²⁹ We take the liberty of not maintaining a clear distinction between instrument leverage within a hedge fund and levered exposure to a hedge fund (such as within a FoHF) since we are only working with returns and not positions.

³⁰ We also assume that the numerators in SR & Treynor are annualized arithmetically to ensure that leverage invariance still holds.

³¹ As before we assume excess returns.

AIRAP only turns negative in going from 10x to 15x (Figure 4). Hence, AIRAP provides risk-adjustment for leverage customized to the investor's risk-aversion. An AIRAP based Sharpe ratio, defined as a function of CRR would also respond to leverage (since the denominator incorporates risk-aversion) though not identically:

$$\text{MSR-AIRAP} = (\text{Excess TR} / \text{RP}(4))$$

The difference is attributable to penalizing for Risk-Premium multiplicatively (in MSR-AIRAP) vis-à-vis additively (in AIRAP).

Finally, the dependence of AIRAP on leverage³² (Table 8), tells both the HF manager and the institutional investor what degree of leverage is optimal for a given track record. Standard optimization techniques (qua first and second order conditions in terms of the partial derivative of AIRAP on leverage) can provide the *optimal leverage*, which *maximizes AIRAP*. Figure 4 shows the AIRAP profile across varying leverage for a range of CRRs. For the growth-optimal case, the Kelly criterion³³ provides the answer.

§6. Hedge fund peer percentile rankings

Realized HF peer rankings within category³⁴ can be directly calculated based on realized AIRAP. However, for a prospective measure that may better handle iceberg risks without the complications of a regime switching implementation, we propose (for future implementation) a composite percentile ranking framework based on a weighted average of the funds style category percentile and stressed scenario percentile. The weights should be fixed from intra-style category testing (e.g., $w_1 = .7$ & $w_2 = .3$) such that:

$$\text{Composite AIRAP \%tile} = \{w_1 * \text{AIRAP Style \%tile} + w_2 * \text{AIRAP Stress \%tile}\},$$

$$\& \text{AIRAP Style \%tile} = 5y \text{ AIRAP \%tile ranking within style category}$$

Given that most HFs have a far shorter history than their traditional counterparts, this may appear to be impractical counsel. However, a number of simulation and optimization techniques have emerged for back-filling history, which can remedy the paucity of available data. Attractive candidates include fitting optimal factor or style exposures to fund profiles based on available history. This will allow one to *extend the style signature back in time* via factor or style exposures that have adequate history. A plethora of multi-

³² While it is better to chart this with the base case being un-levered performance, it is clear that the unit of leverage for the independent variable here (EACM100) is simply a multiple of the leverage already inherent to the EACM100 index.

³³ Kelly, J.L., 1956, "A New Interpretation of Information Rate", Bell Systems Technical Journal, 35, pp. 917-926.

³⁴ Morningstar appears to have recently adopted a similar framework (although their research is not fully in the public domain). Arguably that may not be necessary for mutual fund rankings, but it does provide further validation of the practicality of such an approach. There are also press reports indicating that they are using the Stutzer index.

factor models have been proposed for HFs, e.g., Schneeweis and Spurgin (1998)³⁵ or Fung and Hsieh (1997)³⁶. Further, one can use style analysis - originally proposed by Sharpe (1992) for mutual funds - and applied to HFs³⁷ by Agarwal and Naik (1999) or Fung and Hsieh (1998)³⁸. Indices better known for their style pure classification schema (such as Standard & Poors, EACM or Zurich) should be used to extend backwards the earliest known weighted average *style signature* (assuming no style drift) to facilitate calculation of *AIRAP Style %tile*.

The inclusion of *AIRAP Stress %tile* is warranted due to dormant dangers that may be lurking in the higher moments but not manifest in the 5 year trailing period. Industry consensus is required for establishing representative, preset crash test scenarios encompassing credit, interest rate, volatility and equity events. Obvious candidates for equity include 1987 and 2000, 1994 for fixed income, while 1997 and 1998 may suffice for credit and default scenarios. Incorporating historical crises is critical to capturing higher moment risks, hence potential rank reversals: E.g., The volatility spike resulting from the Russian default dealt swift justice to short volatility players, whose previously pristine track records abruptly realized the dormant dangers of their 'true' risk profile. In fact, using just the three year period [12/99-11/02], which omits these credit and volatility spikes, shows rather different results with Non-Directional strategies displaying dramatically lower kurtosis (even less than directional strategies during this period) and more favorable skew.

³⁵ Schneeweis, T. & Spurgin, R., 1998, "Multifactor Analysis of Hedge Fund, Managed Futures, and Mutual Fund Return and Risk Characteristics", *Journal of Alternative Investments*, 1, 1-24.

³⁶ Fung, W. & Hsieh, D., 1997, "Empirical characteristics of dynamic trading strategies: the case of hedge funds", *The Review of Financial Studies*, 10:275-302.

³⁷ Weisman and Abernathy (2001) propose a non-parametric alternative via Generic Model Decomposition. However, their approach requires subjective judgement in the choice of variables for each fund on a case by case basis. See Weisman, A. & Abernathy, J. D., 2001, "The Dangers of Historical Hedge Fund Data", *Risk Budgeting – A New Approach to Investing*, Edited by Leslie Rahl, Risk Books.

³⁸ Fung, W. & Hsieh, D., 1998, "Performance Attribution and Style Analysis: From Mutual Funds to Hedge Funds," working paper.

§7. Caveats & Conclusion

AIRAP presents a radical departure from preference free RAPMs in circulation. At the same time, it benefits from the familiar and established lineage of Expected Utility theory. Salient features of AIRAP, which enhance its suitability as a RAPM for hedge funds include:

- Appropriate treatment of leverage for hedge funds
- Distribution free framework eschews unrealistic assumption of Normality
- Incorporation of investor preferences via Power utility, which given CRRA is more realistic than Quadratic utility underlying Mean-Variance. Risk adjustment is not ad-hoc, nor does it misrepresent upside risk. Downside variance is penalized more.
- AIRAP better handles non-normality since it directly utilizes the full empirical distribution. Unlike higher order approximations (e.g. MSR based on a Cornish-Fisher modified VaR expansion), there is no sacrifice in accuracy due to the truncation of higher order terms.
- Scale invariance of CRRA inherent to AIRAP
- Consistent rankings even when mean excess returns are negative³⁹
- Intuitively expressed in familiar units of performance
- AIRAP maximization is equivalent to maximizing EU. Hence, it can be utilized for portfolio optimization as in the case of FoHFs.
- AIRAP can better handle Non-Directional/ Market-Neutral strategies
- AIRAP can be expressed as a modified SR⁴⁰ to preserve the reward-risk format
- No complications regarding the estimation of higher moments, co-moments or convergence of Taylor series
- AIRAP can dovetail with regime switching models or be combined with scenario stresses, for handling iceberg risks. While regime-switching models provide a systematization of the ad-hoc scenario analysis prevalent in practice, they do require regime identification and technical complexities that may present barriers to practicability.
- Possible to use closed form solution with easy spreadsheet implementation.

Traditional portfolio construction of FoHFs based on SR maximization can result in a bias towards illiquidity and short volatility. Measures such as AIRAP that mitigate the vulnerabilities of SR can help circumnavigate the dangers lurking in higher moments. As FoHF portfolio construction usually entails a 2-step top-down procedure where the optimal style weights are determined before individual manager weights, refining the first optimization (by transcending the mean-variance framework) should help in avoiding the pitfalls of improperly weighting styles. Getting the style allocation decision right also means that the FoHF manager can focus more on the ‘selection’ challenge of picking the right managers and performing the necessary due diligence to avoid operational risk or fraud. We have demonstrated the criticality of AIRAP to the ‘selection’ challenge via better rankings. AIRAP as presented in this paper maximizes ease of practical use at the

³⁹ Given two portfolios with the same negative mean excess return, SR will erroneously rank the one with higher risk (hence less negative ratio) as the better portfolio.

⁴⁰ However, MSR-AIRAP inherits some of the disadvantages of SR.

stand-alone fund level. We leave the application of EU theory towards FoHF portfolio construction using marginal considerations and correlations with other investments as fodder for future research.

Effects such as putatively managed or stale pricing⁴¹ may also be masking the true statistical properties. Lo (2002) and Getmansky et al. (2003) have documented the extent of serial correlation observed in HF returns and its upward bias on RAPMs like SR. Hence, it would be interesting to apply AIRAP to un-smoothed returns since it would adjust for illiquidity/ stale pricing in addition to higher moment risks. To the extent that survivorship would likely bias means and skews upwards while depressing the true volatilities and kurtoses, it appears that even if one were to adjust for survivorship, the divergence between AIRAP and SR or JA reported here would only be exacerbated. Although SR would also drop given the mean-variance impact it may not be impacted as much as AIRAP upon incorporation of higher moments.

Meantime, a healthy debate on the ‘near adequacy’ of SR and the mean-variance framework continues. Barring ex-ante prescience, it appears that one should err on the side of caution by also considering RAPMs such as AIRAP which stand a better chance of survival in stressed scenarios.

⁴¹ See Asness, C., R. Krail and J. Liew, 2001, “Do Hedge Funds Hedge?” *Journal of Portfolio Management* , 28(1), 6-19.

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Table 1.

S&P 500 mo returns	Excess Kurt	Skew	Vol	Av Monthly return
40 yrs [2/62-1/02]	1.91	(0.32)	4.33	0.97
30 yrs [2/72-1/02]	2.20	(0.36)	4.49	1.06
20 yrs [2/82-1/02]	2.96	(0.67)	4.43	1.29
10 yrs [2/92-1/02]	1.10	(0.66)	4.04	1.10

Table 2.

GoF Tests	40 YRs - 2 SIDED Goodness of Fit			
	Asympt Pval	Test stat	95% Critical Value	99% Critical Value
Lilliefors	0.0486*	0.0408	0.0404	0.0503
Bera-Jarque	- * **	77.2904	5.9915	9.2103

* & ** correspond to 95% & 99% levels of significance

Table 3. Sharpe ratios (Negative mean returns)

EACM 5yrs[97-01]	Bond * Hedge	FI hedge fund
Ann vol	6.02%	12.04%
Ann %Excess TR	-4.50%	-4.50%
Ann Sharpe	(0.75)	(0.37)

Table 4. RAPM summary - HFR universe [787 funds, 1997-2001]

	Average	Median	Min	Max
EXTR	6.53%	6.01%	-25.10%	44.76%
Vol	16.55%	13.83%	0.12%	100.07%
Skew mo	(0.14)	(0.01)	(7.18)	5.78
Excess Kurt mo	3.02	1.28	(0.86)	51.41
Treynor	0.05	0.19	(60.66)	38.93
Alpha	6.2%	5.6%	-24.4%	66.5%
Beta	0.29	0.20	(1.75)	2.12
Sharpe	0.75	0.57	(1.72)	7.54
AIRAP	-0.02%	2.99%	-93.25%	25.63%
MSR	13.65	2.13	(74.00)	1,600.14

Table 4 shows the aggregate statistics for the 1st four moments and various RAPMs based on 787 individual hedge funds in the HFR universe for the 5 year period

Table 5. EACM Sub-Indices		Relative Value				Event Driven			Equity Hedge Fds			Global AA		Shorts	Index
Summary Stats - 5yrs [97-01]		Long/Short	Convertible	Bond	Multi-Strategy	Risk	Bankruptcy/	Multi-	Domestic	Domestic	Global/			Short	
		Equity	Hedge	Hedge		Arbitrage	Distressed	Strategy	Long Bias	Opportunistic	International	Discretionary	Systematic	Sellers	SP500
Mean ExTR mo	0.00%	0.32%	-0.37%	0.73%	0.35%	0.29%	0.73%	0.66%	1.06%	0.58%	0.19%	0.19%	0.25%	0.59%	
Median ExTR mo	0.09%	0.50%	0.15%	0.97%	0.49%	0.51%	0.86%	0.35%	0.85%	0.66%	0.03%	-0.16%	-0.46%	0.71%	
Vol ExTR Ann	2.98%	5.99%	6.02%	8.44%	4.87%	6.22%	6.18%	21.93%	12.91%	13.05%	10.19%	12.06%	22.78%	17.91%	
Skew ExTR mo	(0.56)	(2.11)	(2.12)	(4.61)	(2.49)	(1.87)	(2.31)	(0.06)	1.03	0.22	(2.60)	0.62	0.72	(0.54)	
Excess Kurt ExTR mo	1.65	6.28	5.76	24.70	9.06	8.32	11.49	0.23	2.14	0.89	14.90	1.15	0.47	(0.16)	
Ann Sharpe	0.02	0.65	(0.73)	1.04	0.86	0.56	1.41	0.36	0.99	0.54	0.22	0.19	0.13	0.39	
Ann Treynor	(0.10)	0.60	(1.24)	2.20	0.31	0.22	0.57	0.10	0.62	0.16	0.07	(0.44)	(0.03)	0.07	
Ann Jensen's Alpha	0.09%	3.40%	-4.67%	8.51%	3.24%	2.34%	7.64%	2.04%	11.28%	3.92%	-0.01%	2.65%	10.55%	0.00%	
CAPM Beta	(0.01)	0.06	0.04	0.04	0.13	0.16	0.15	0.83	0.20	0.44	0.32	(0.05)	(1.08)	1.00	
Corr to S&P500	(0.03)	0.19	0.10	0.08	0.49	0.46	0.45	0.68	0.28	0.60	0.56	(0.08)	(0.84)	1.00	
Lilliefors GoF test (2-sided, 95%)	0	1	1	1	1	1	1	0	0	0	1	0	0	0	
Bera-Jarque GoF test (2-sided, 95%)	1	1	1	1	1	1	1	0	1	0	1	1	1	1	
P value Lilliefors (2-sided, 95%)	0.10	NaN	NaN	NaN	NaN	0.04	NaN	NaN	NaN	NaN	NaN	0.11	NaN	0.07	
P value Bera-Jarque (2-sided, 95%)	0.05	-	-	-	-	-	-	0.97	0.00	0.37	-	0.04	0.07	0.07	
Ann ExTR	0.01%	3.75%	-4.50%	8.75%	4.14%	3.32%	8.89%	5.66%	12.60%	6.37%	1.72%	1.59%	0.46%	5.61%	
Ann AIRAP [CRR=4]	-0.13%	3.16%	-5.06%	7.32%	3.75%	2.69%	8.23%	-1.71%	10.08%	3.77%	-0.14%	-0.49%	-6.42%	0.45%	
Ann Risk Prem [CRR=4]	0.13%	0.59%	0.55%	1.43%	0.39%	0.63%	0.66%	7.37%	2.52%	2.59%	1.86%	2.08%	6.88%	5.16%	
Modified SR	0.05	6.38	(8.12)	6.12	10.66	5.27	13.46	0.77	5.01	2.45	0.93	0.76	0.07	1.09	

AIRAP vs SR [#Reversals, Total#, %Rev]

11 13 **85%**

MSR vs SR [#Reversals, Total#, %Rev]

7 13 **54%**

* For GoF tests 1 corresponds to REJECTION of Normality

* Bera-Jarque: The Null assumes normality with standardized skew & kurtosis being asymptotically normal and independent

* Lilliefors: Modifies Kolmogorov-Smirnov. Test statistic used is max|Empirical CDF - Gaussian CDF|. P-values outside [0.01, 0.20] are reported as NaN

* All stats are annualized & based on Excess TR

Spearman Correlations	ExTR	Vol	Skew mo	Excess Kurt mo	Treynor	Jensen	Beta	Sharpe	AIRAP	MSR
ExTR	1.00	0.23	(0.05)	0.24	0.85	0.65	0.60	0.90	0.84	0.74
Vol	0.23	1.00	0.57	(0.53)	(0.06)	0.37	0.27	(0.07)	(0.20)	(0.25)
Skew mo	(0.05)	0.57	1.00	(0.85)	(0.25)	0.31	(0.07)	(0.30)	(0.19)	(0.43)
Excess Kurt mo	0.24	(0.53)	(0.85)	1.00	0.49	(0.01)	0.05	0.54	0.48	0.60
Treynor	0.85	(0.06)	(0.25)	0.49	1.00	0.66	0.37	0.94	0.84	0.86
Jensen	0.65	0.37	0.31	(0.01)	0.66	1.00	(0.06)	0.63	0.57	0.47
Beta	0.60	0.27	(0.07)	0.05	0.37	(0.06)	1.00	0.39	0.37	0.35
Sharpe	0.90	(0.07)	(0.30)	0.54	0.94	0.63	0.39	1.00	0.87	0.93
AIRAP	0.84	(0.20)	(0.19)	0.48	0.84	0.57	0.37	0.87	1.00	0.77
MSR	0.74	(0.25)	(0.43)	0.60	0.86	0.47	0.35	0.93	0.77	1.00
2-Sided Correlation p-values	ExTR	Vol	Skew mo	Excess Kurt mo	Treynor	Jensen	Beta	Sharpe	AIRAP	MSR
ExTR	-	0.45	0.87	0.43	0.00	0.02	0.03	0.00	0.00	0.00
Vol	0.45	-	0.04	0.06	0.84	0.21	0.36	0.82	0.51	0.42
Skew mo	0.87	0.04	-	0.00	0.40	0.31	0.82	0.32	0.54	0.14
Excess Kurt mo	0.43	0.06	0.00	-	0.09	0.97	0.87	0.06	0.09	0.03
Treynor	0.00	0.84	0.40	0.09	-	0.01	0.22	0.00	0.00	0.00
Jensen	0.02	0.21	0.31	0.97	0.01	-	0.84	0.02	0.04	0.10
Beta	0.03	0.36	0.82	0.87	0.22	0.84	-	0.19	0.22	0.24
Sharpe	0.00	0.82	0.32	0.06	0.00	0.02	0.19	-	0.00	0.00
AIRAP	0.00	0.51	0.54	0.09	0.00	0.04	0.22	0.00	-	0.00
MSR	0.00	0.42	0.14	0.03	0.00	0.10	0.24	0.00	0.00	-
Ascending ranks by RAPM	ExTR	Vol	Skew mo	Excess Kurt mo	Treynor	Jensen	Beta	Sharpe	AIRAP	MSR
L/S Eq	2	1	8	5	3	3	3	2	6	2
Converts	7	3	6	8	11	8	6	9	8	11
Bond Hedge	1	4	5	7	1	1	4	1	2	1
RV Multi-Strategy	11	7	1	13	13	11	5	12	11	10
Risk Arb	8	2	3	10	9	7	7	10	9	12
Bankruptcy/ D	6	6	7	9	8	5	9	8	7	9
ED Multi-Strategy	12	5	4	11	10	10	8	13	12	13
Dom Long	9	12	9	1	6	4	13	6	3	5
Dom Opp	13	10	13	6	12	13	10	11	13	8
Global International	10	11	10	3	7	9	12	7	10	7
GAA Discretionary	5	8	2	12	5	2	11	5	5	6
GAA Systematic	4	9	11	4	2	6	2	4	4	4
Shorts Sellers	3	13	12	2	4	12	1	3	1	3

Table 6: RAPM stats - HFR universe [787 funds, 1997-2001]

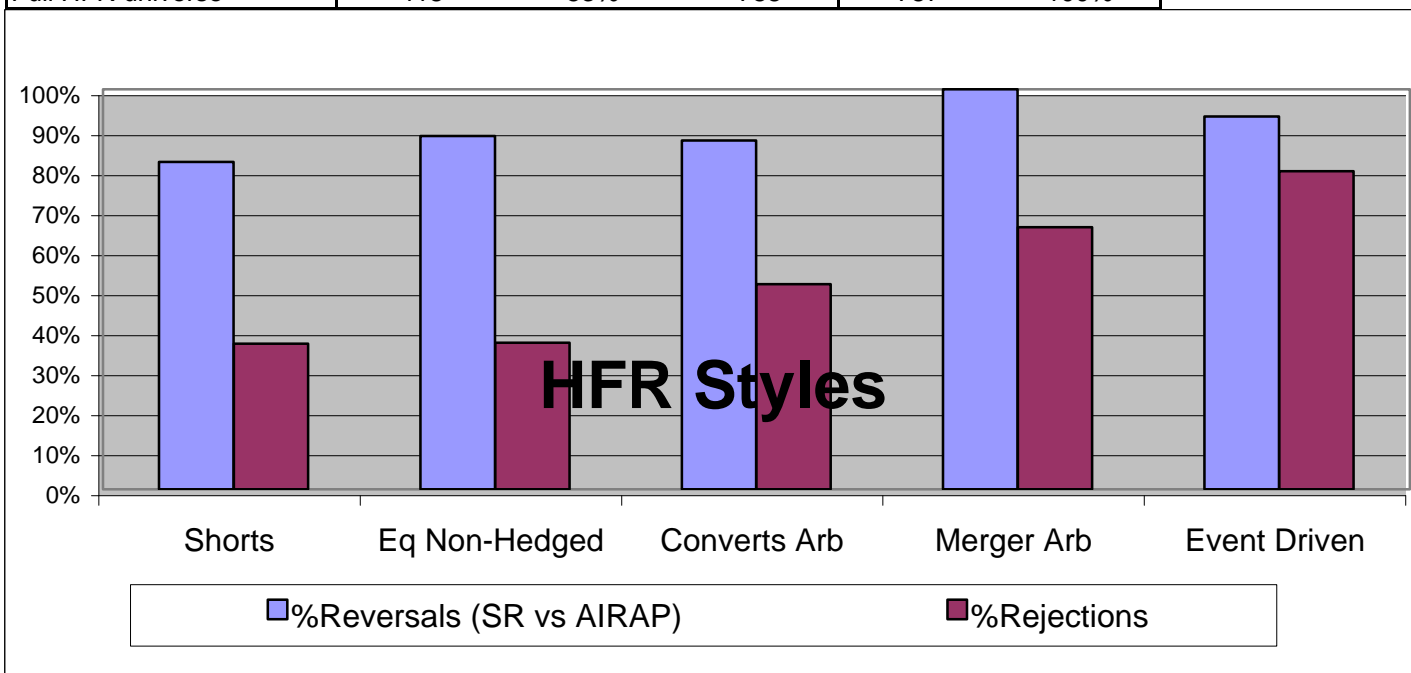
Spearman Correlations	ExTR	Vol	Skew mo	Excess Kurt mo	Treynor	Jensen	Beta	Sharpe	AIRAP	MSR
ExTR	1.00	0.06	0.26	0.02	0.41	0.90	0.08	0.73	0.76	0.49
Vol	0.06	1.00	0.22	(0.13)	(0.32)	0.11	0.55	(0.50)	(0.47)	(0.73)
Skew mo	0.26	0.22	1.00	(0.18)	0.11	0.32	(0.03)	0.07	0.14	(0.02)
Excess Kurt mo	0.02	(0.13)	(0.18)	1.00	0.10	0.04	(0.05)	0.12	0.08	0.14
Treynor	0.41	(0.32)	0.11	0.10	1.00	0.34	0.04	0.60	0.50	0.57
Jensen	0.90	0.11	0.32	0.04	0.34	1.00	(0.16)	0.66	0.66	0.43
Beta	0.08	0.55	(0.03)	(0.05)	0.04	(0.16)	1.00	(0.23)	(0.24)	(0.37)
Sharpe	0.73	(0.50)	0.07	0.12	0.60	0.66	(0.23)	1.00	0.86	0.92
AIRAP	0.76	(0.47)	0.14	0.08	0.50	0.66	(0.24)	0.86	1.00	0.80
MSR	0.49	(0.73)	(0.02)	0.14	0.57	0.43	(0.37)	0.92	0.80	1.00
Significance	ExTR	Vol	Skew mo	Excess Kurt mo	Treynor	Jensen	Beta	Sharpe	AIRAP	MSR
ExTR	-	0.11	0.00	0.57	-	-	0.03	-	-	-
Vol	0.11	-	0.00	0.00	-	0.00	-	-	-	-
Skew mo	0.00	0.00	-	0.00	0.00	-	0.44	0.05	0.00	0.53
Excess Kurt mo	0.57	0.00	0.00	-	0.01	0.29	0.17	0.00	0.02	0.00
Treynor	-	-	0.00	0.01	-	-	0.23	-	-	-
Jensen	-	0.00	-	0.29	-	-	0.00	-	-	-
Beta	0.03	-	0.44	0.17	0.23	0.00	-	0.00	0.00	-
Sharpe	-	-	0.05	0.00	-	-	0.00	-	-	-
AIRAP	-	-	0.00	0.02	-	-	0.00	-	-	-
MSR	-	-	0.53	0.00	-	-	-	-	-	-

Stats/ RAPMs	Mean ExTR	Median ExTR	Vol	Skew	Excess Kurt	SR	Treynor	Jensen	Beta (S&P)	ExTR	AIRAP	Risk Prem	MSR
Pearson Correl (AIRAP)	0.22	0.12	(0.74)	0.06	(0.09)	0.46	(0.01)	0.37	(0.38)	0.62	1.00	(0.82)	0.06
Mean	0.69%	0.58%	16.55%	(0.14)	3.02	0.75	0.05	6.22%	0.29	6.53%	-0.02%	6.55%	13.65
Standard Error	0.02%	0.03%	0.46%	0.04	0.19	0.03	0.16	0.28%	0.02	0.28%	0.49%	0.39%	3.05
Median	0.59%	0.53%	13.83%	(0.01)	1.28	0.57	0.19	5.64%	0.20	6.01%	2.99%	2.97%	2.13
Mode	0.78%	0.75%	46.06%	(0.37)	4.00	0.09	(0.03)	14.36%	(1.45)	-7.19%	-46.41%	39.22%	(0.18)
Standard Deviation	0.66%	0.73%	12.87%	1.23	5.39	0.82	4.43	7.93%	0.46	7.92%	13.78%	10.82%	85.69
Sample Variance	0.00%	0.01%	1.66%	1.50	29.08	0.67	19.60	0.63%	0.21	0.63%	1.90%	1.17%	7,341.92
Kurtosis	4.87	3.77	4.98	5.89	28.46	14.57	103.59	5.71	2.26	2.19	10.70	22.44	235.44
Skewness	1.20	(0.03)	1.78	(1.24)	4.54	2.68	(6.82)	0.84	0.32	0.23	(2.73)	4.12	14.70
Minimum	-1.54%	(0.03)	0.00	(7.18)	(0.86)	(1.72)	(60.66)	-24.36%	(1.75)	(0.25)	(0.93)	0.00	(74.00)
Maximum	5.25%	0.04	1.00	5.78	51.41	7.54	38.93	66.53%	2.12	0.45	0.26	0.89	1,600.14

AIRAP is positively correlated with ExTR, skew, Treynor, Jensen, SR & negatively with Vol & Beta as per intuition
 All AIRAP correlations are highly significant

Table 7. %Rejections of Normality vs %Rank reversals

HFR Intra-Style (97-01)	#Rejections	%Rejections	#Funds Tot	#Reversals	%Reversals
Shorts	4	36%	11	9	82%
Eq Non-Hedged	22	37%	60	53	88%
Converts Arb	20	51%	39	34	87%
Merger Arb	19	66%	29	29	100%
Event Driven	35	80%	44	41	93%
Full HFR universe	418	53%	788	787	100%



Rejections of Normality are based on Bera-Jarque GoF test (2-sided, 95%)
 Reversals are rank reversals between Sharpe and AIRAP

Table 8. Change in RAPMs vs Change in Leverage

Leverage factor	2	5	10	15	Response
Leverage increase	2.00	2.50	2.00	1.50	
ExTR arith	2.00	2.50	2.00	1.50	Linear
Vol	2.00	2.50	2.00	1.50	Linear
Skew	1.00	1.00	1.00	1.00	Invariant
Excess Kurtosis	1.00	1.00	1.00	1.00	Invariant
Sharpe	1.00	1.00	1.00	1.00	Invariant
Treynor	1.00	1.00	1.00	1.00	Invariant
Alpha	2.00	2.50	2.00	1.50	Linear
Beta	2.00	2.50	2.00	1.50	Linear
AIRAP	1.76	1.35	-1.83	4.78	Non Linear
ExTR geom	1.98	2.40	1.76	1.14	Non Linear
RiskPrem (Arith)	4.15	6.98	4.88	2.43	Non Linear

Table 9. Representative fund RAPM comparisons

Fund ID	AIRAP	Sharpe	Treynor	Jensen	Beta	ExTR	Vol	Skew	ExKurt
229	13	482	62	788	25	787	787	719	590
230	11	362	302	753	786	679	781	398	507
231	10	420	351	777	783	747	780	373	550
235	202	1	48	65	176	69	8	423	74
272	2	234	256	781	655	223	788	788	786
512	1	133	151	81	771	10	776	58	682
636	788	699	599	776	635	784	545	667	361
762	373	788	784	221	143	201	2	485	174

Fund ID	AIRAP	Sharpe	Treynor	Jensen	Beta	ExTR	Vol	Skew mo	ExKurt
229	-48.57%	0.76	(1.27)	66.53%	(0.50)	37.92%	83.16%	1.13	3.23
230	-51.15%	0.53	0.19	20.27%	1.75	14.09%	61.08%	0.01	2.27
231	-51.64%	0.62	0.23	25.93%	1.64	20.12%	60.23%	(0.05)	2.68
235	-2.76%	-1.72	(1.40)	-2.88%	0.02	-2.72%	1.60%	0.07	(0.10)
272	-86.14%	0.34	0.49	29.23%	0.69	3.03%	100.07%	5.78	41.15
512	-93.25%	0.15	0.06	-2.09%	1.45	-13.39%	56.17%	(1.82)	6.00
636	25.63%	1.54	0.48	25.40%	0.62	31.96%	19.32%	0.80	1.13
762	2.41%	7.54	9.38	2.37%	0.00	2.41%	0.32%	0.22	0.34

Table 10. AIRAP, Sharpe & Jensen Alpha as a function of Risk-Aversion

CRR	#Reversals SR*	#Reversals JA*	%Reversals SR	%Reversals JA	AIRAP vs SR	AIRAP vs Alpha	F&H'97
0.1	781	779	99%	99%	0.59	0.89	0.49
0.2	780	775	99%	98%	0.61	0.89	0.50
0.3	784	777	99%	99%	0.63	0.90	0.52
0.4	783	776	99%	98%	0.64	0.90	0.53
0.5	786	776	100%	98%	0.66	0.91	0.55
1	783	772	99%	98%	0.73	0.90	0.52
1.5	784	783	99%	99%	0.78	0.86	0.68
2	787	785	100%	100%	0.82	0.82	0.73
2.5	785	784	100%	99%	0.84	0.78	0.77
3	785	784	100%	99%	0.86	0.74	0.81
3.5	786	783	100%	99%	0.86	0.70	0.84
4	787	786	100%	100%	0.86	0.66	0.85
4.5	786	786	100%	100%	0.87	0.63	0.87
5	783	787	99%	100%	0.87	0.60	0.89
10	785	784	100%	99%	0.80	0.34	0.89
15	788	786	100%	100%	0.74	0.23	0.87
20	788	786	100%	100%	0.70	0.17	0.85
25	784	787	99%	100%	0.68	0.14	0.83
30	786	787	100%	100%	0.66	0.11	0.81

AIRAP vs Sharpe & Jensen represent Spearman rank correlations

Correlation with Jensen tapers off rapidly

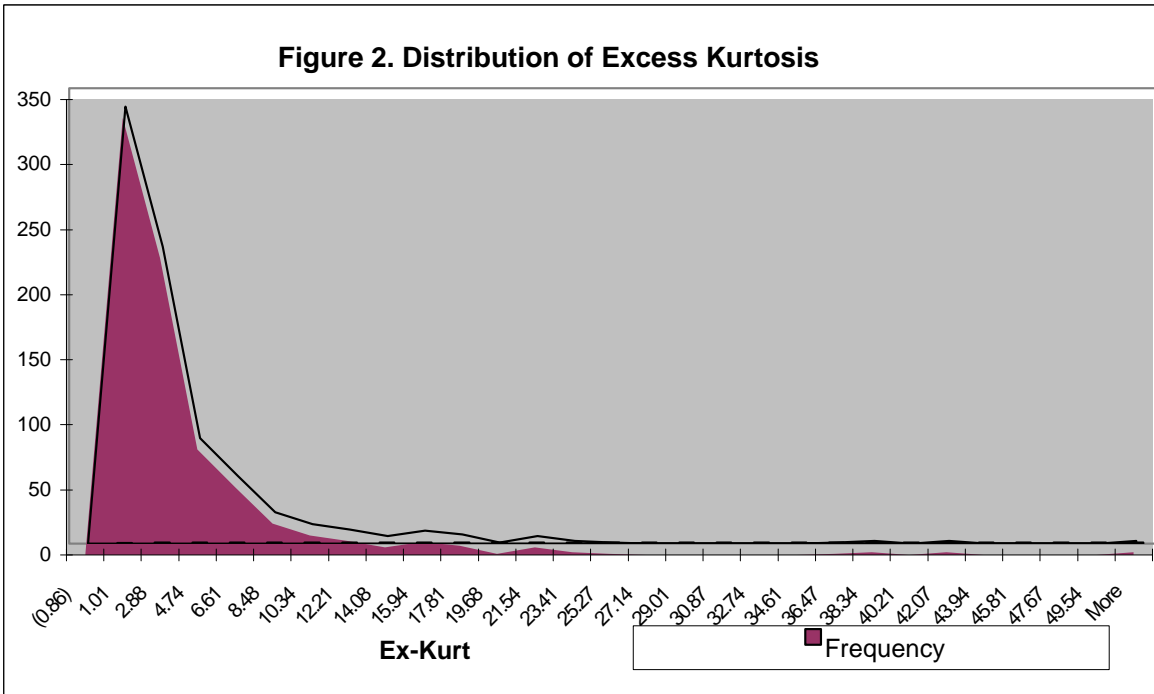
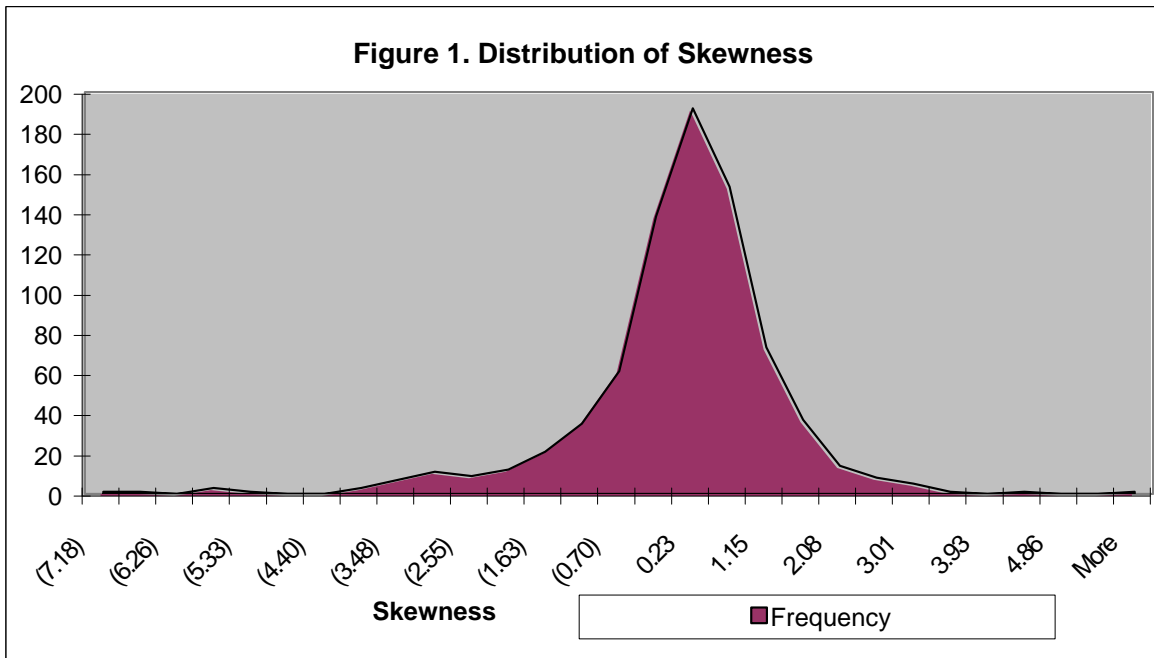
F&H'99: Rank correlations of [power utility, SR] from Fung & Hsieh (1999)

%Reversals SR show nearly 100% rank reversals between Sharpe & AIRAP

%Reversals JA show nearly 100% rank reversals between Jensen & AIRAP

Data: 787 funds in HFR for the period 1997-2001

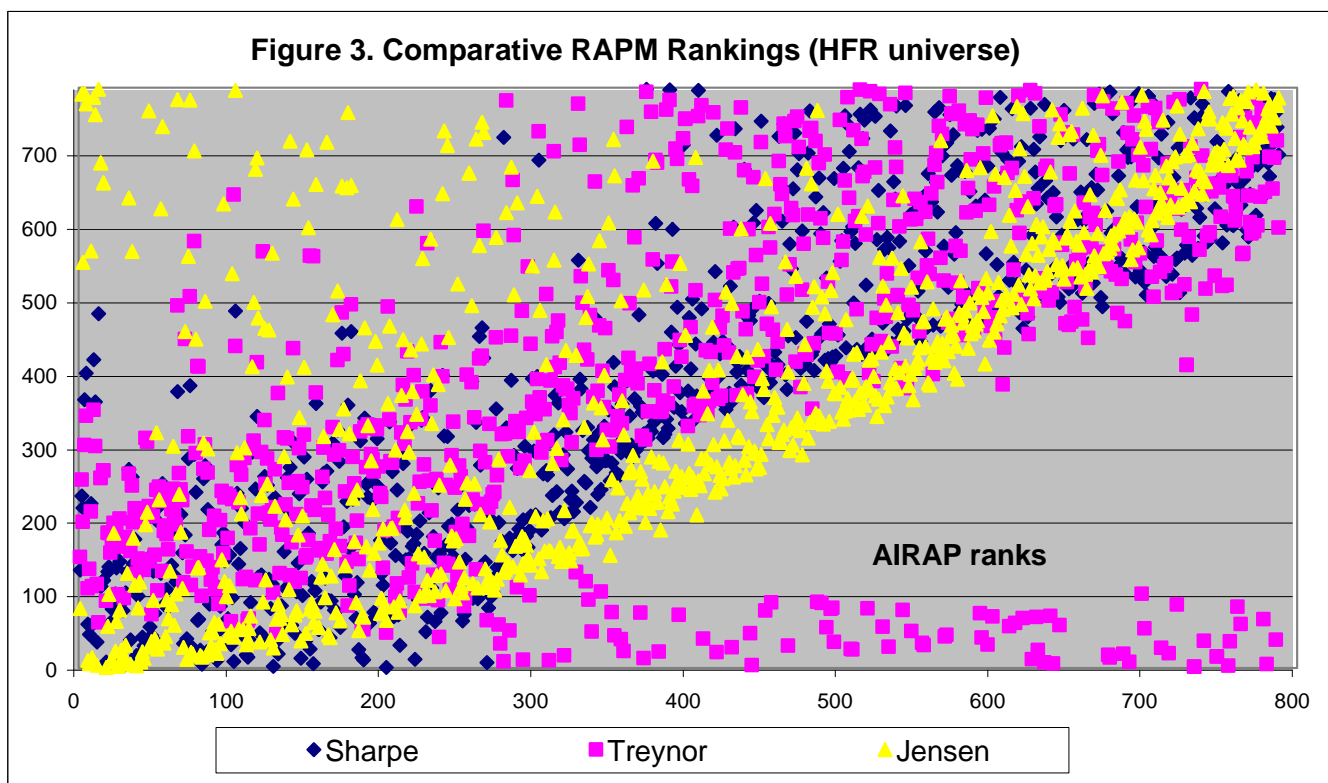
Source: Hedge Fund Research, Inc., © HFR, Inc., www.hedgefundresearch.com



The distribution of excess kurtosis for hedge funds during the 5 years (01/97-12/01) is clearly right skewed (+4.54) with a long right tail (max of 51.4 but min of -.86). Average Ex-Kurt of 3.0 is significantly non-Gaussian with 87.4% of all funds in positive territory.

The distribution of HF skewness shows mild negative skew of -1.24 apparently due to the counterbalancing effect of including CTAs. Still the left tail is longer given min of -7.18 vs max of 5.78 while mean, median & mode are all negative.

Source for 787 HFs used is Hedge Fund Research, Inc., © HFR, Inc., www.hedgefundresearch.com



All RAPM ranks are in ascending order with higher ranks being more desirable

AIRAP (CRR=4) rankings are shown on the x-axis

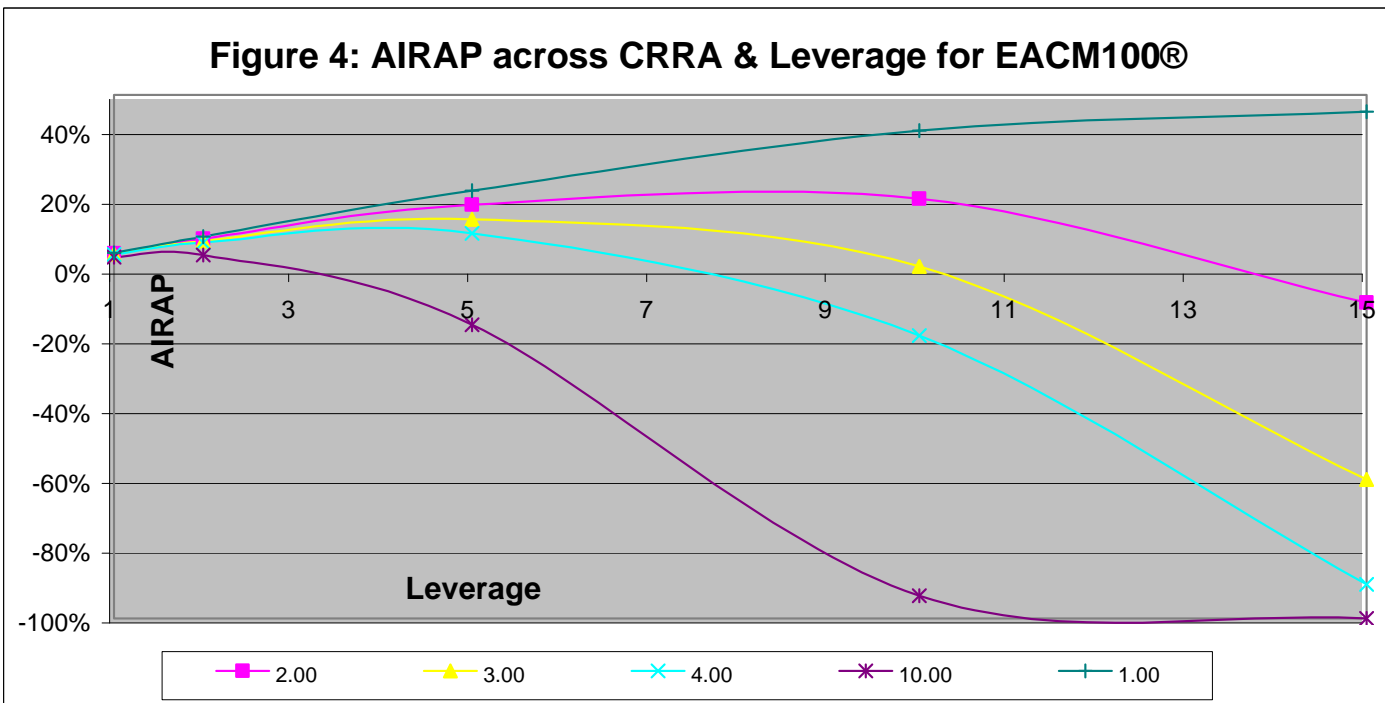
The abundance of off-diagonal data shows the extent of divergence between the 3 RAPMS vis-à-vis AIRAP

The cluster of pyramids in the top left represents high JA funds demoted by AIRAP

The cluster of squares at the bottom right represents high AIRAP funds demoted by Treynor

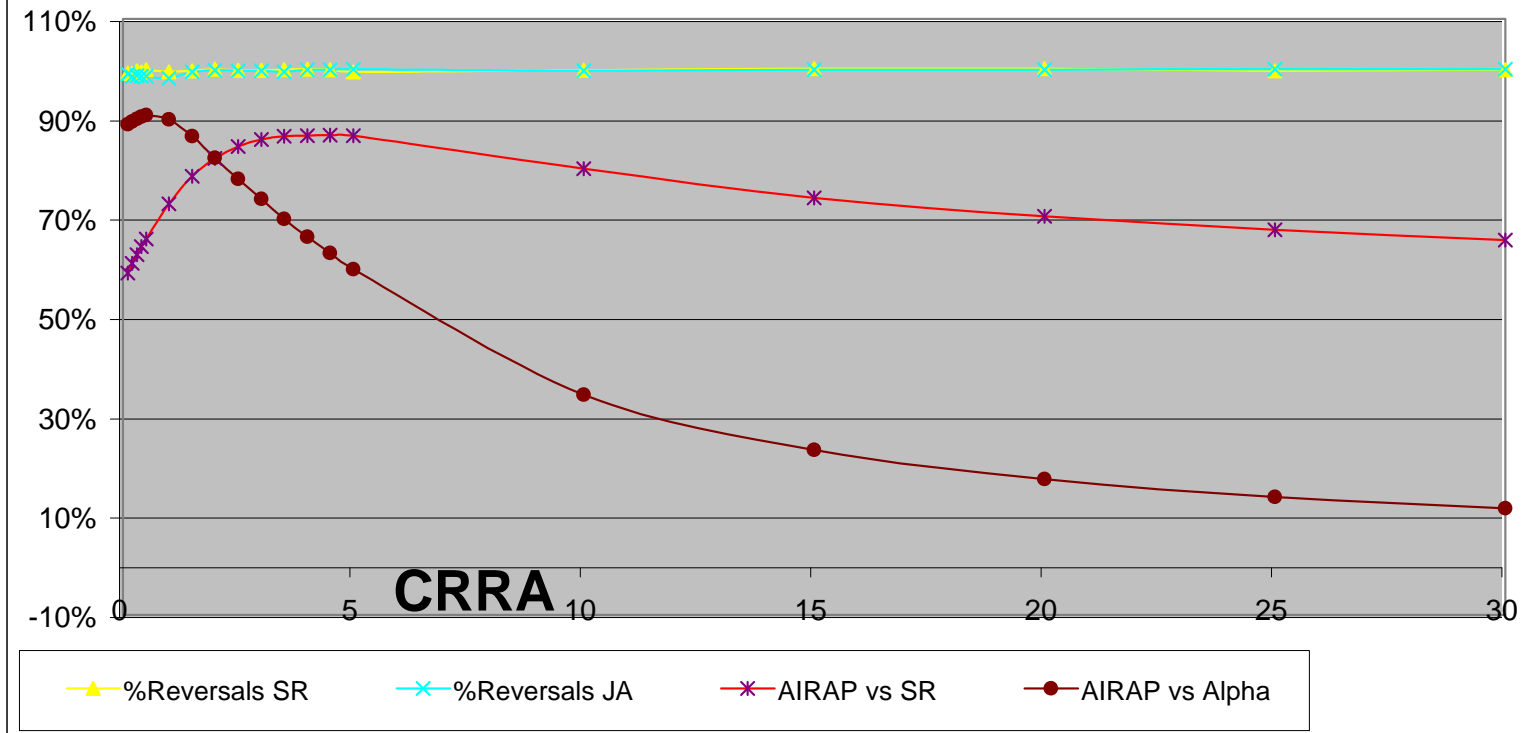
Source for 787 HFs used is Hedge Fund Research, Inc., © HFR, Inc., www.hedgefundresearch.com

Figure 4: AIRAP across CRRA & Leverage for EACM100®



CRRA/ Leverage	1	2	5	10	15
1.00	4.77%	9.44%	22.63%	39.85%	45.23%
2.00	4.62%	8.84%	18.48%	20.32%	-9.44%
3.00	4.48%	8.24%	14.39%	0.86%	-60.10%
4.00	4.33%	7.64%	10.30%	-18.89%	-90.27%
10.00	3.47%	4.07%	-15.83%	-93.48%	-100.00%

Figure 5. % Reversals & Rank Correlations by Risk Aversion



AIRAP vs Sharpe & Jensen represent Spearman rank correlations

Correlation with Jensen tapers off rapidly

%Reversals SR show nearly 100% rank reversals between Sharpe & AIRAP

%Reversals JA show nearly 100% rank reversals between Jensen & AIRAP

Data: 788 funds in HFR for the period 1997-2001

Source: Hedge Fund Research, Inc., © HFR, Inc., www.hedgefundresearch.com

Figure 6. AIRAP (Certainty Equivalent for CRRA=4) under Risk-Aversion

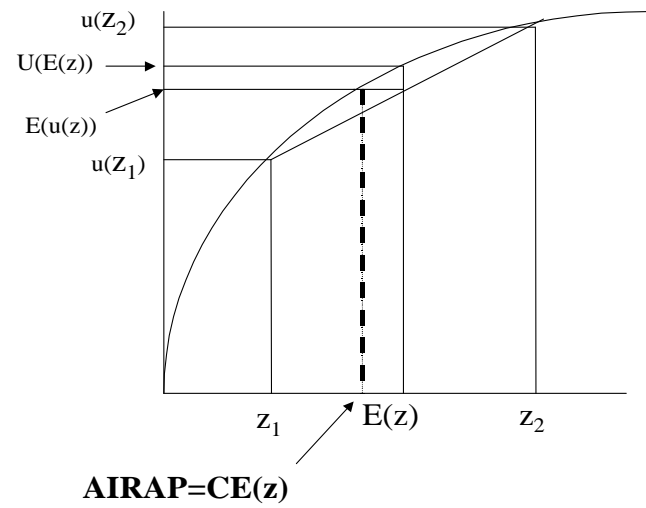


Figure 7. RAPMs vs Leverage (CRR=4) - EACM 100

